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ABSTRACT

This paper is a report of a 1974 conference of Teachers of Teachers of Mathematics (TOTOM) from Oregon colleges. The conference was organized for the purpose of discussing competencies of elementary teachers of mathematics. The conferees took the position that they could not define necessary and sufficient criteria for the competent teaching of mathematics, but that they could outline a set of goals and expectancies. Expectancies were developed in three major areas: problem solving, pedagogy, and mathematical content. In the discussion of problem solving, the works of several writers are considered, and several aspects of the process of problem solving are identified; 30 specific competencies related to problem solving are listed. In the area of pedagogy expectancies are listed in four major areas (psychology of learning, diagnosis, resources and materials, and teaching strategies); sample behaviors are listed for each expectancy. Expectancies and related sample behaviors are listed for six mathematical content areas (sets, operations, number sentences, number theory, logical thinking, and geometry). (SD)

SUGGESTED EXPECTANCIES FOR ELEMENTARY

TEACHERS OF MATHEMATICS

A Working Paper

for the

SECOND ANNUAL TOTOM CONFERENCE
MARYLHURST EDUCATION CENTER

September 7-9, 1975

Co-Hosted and Co-Funded by

OREGON DEPARTMENT OF EDUCATION

AND

OREGON SYSTEM OF MATHEMATICS EDUCATION

INTRODUCTION

During the 1972-73 school year a group of mathematics education leaders began work on a paper listing mathematics competencies for elementary teachers. The group included Jim Barnard, Mildred Bennett, Jay Greenwood, Vern Hiebert, Alan Hoffer, Gary Musser, Ocsar Schaaf and Charles White, with Jay serving as chairman. The paper that group developed, "Competencies for Elementary Teachers of Mathematics," was the basis of discussion for a September 1974 conference. Participants were Teachers of Teachers of Mathematics (TOTOM) from Oregon colleges, universities and community colleges involved in the preparation of elementary teachers.

The Conferees of that first annual TOTOM Conference found that in many respects the questions relating to a competency based teacher education statement are inseparable from the issues of competency-based teacher education. An effort to resolve this dilemma resulted in drafting a statement, offered here, in an attempt to define the role of the TOTOM Conferences.

PREAMBLE - TOTOM 1 CONFERENCE RESOLUTION

We, as mathematics educators, believe that it is not possible to make explicit <u>the</u> criteria that are necessary and sufficient to produce a competent teacher. On the other hand, we as professionals, do have some intuitive notions of what "good" teaching is and some convictions as to the content and methods most appropriate for helping our students become competent teachers.

One function of conferences such as the first annual TOTOM Conference is to explicitly identify as many of these "ingredients" as we can and to communicate this knowledge to our colleagues. Another function is to come up with a common list of "ingredients" which can be used as a nucleus of a mathematics education program in our own teacher education institutions. Whether this list defines a competent teacher is an open question.

PURPOSES OF A COMPETENCY (EXPECTANCY) PAPER

1. Such a paper represents an attempt by educators to put into words that which they hope to accomplish. Therefore, the paper cannot represent a finished product, indeed, it can never be "finished," but represents an evolving definition of goals - not a course outline or a "minimum list." As such, it allows for interaction among colleagues as they work together in developing their own school statement of program goals and expectancies for themselves and their students to see and to use as a guide. From this base, their task then becomes planning and providing spiraled experiences offering learning opportunities leading toward the ststed goals.

2. An inherent deficiency must be heeded. The mere act of writing a paper of expectancies creates some deficiencies. For example, "pedagogy" and "content" are separated here solely for the purpose of convenience in the writing effort. In practice they must be integrated and coordinated, each helping to support the other. Both should utilize the problem solving approach, which appears in this paper as though it were a separate topic. Certain common themes should exist throughout all the learning experiences for the prospective teacher-motivation for learning and teaching mathematics, developing appreciation for mathematics, a friendliness for numbers, and inquisitive attitude - - - at the problem solving level!

3. This Expectancy Paper may serve as a source of information to the Teacher Standards and Practices Commission regarding teacher certification - - - not as a statewide program, but as a guide to the individual schools from which they interpret their respective programs to TSPC. As such a base, the Expectancies may represent goals to be achieved by the time Standard Certification is reached.

4. This paper is not intended as a statement of position on Competency-Based Teacher Education or Performance-Based Teacher Education.

TASKS

1. Identify content areas (ingredients)

2. Identify items within content areas as well as their levels of specificity.

3. Identify additional items which could maximize the usefulness of our final document.

4. Recommend procedures for preparing the document.

5. How will you use the document?

6. Who should receive final copies?

7. What are other issues and trends facing us?

PROBLEM SOLVING

During the first TOTOM Conference there seemed to be general agreement that the major goal of mathematics instruction in schools should be the improvement of the student's ability to solve problems. Therefore, the problem-solving processes should be given much attention in mathematics teacher education programs. However, many participants at the conference felt that the area of problem solving in the first report was given a scanty three-page treatment while eighteen pages were devoted to other matters. There also seemed to be general consensus that the term problem solving needed to be better defined.

The TOTOM 2 Committee members felt that the problem-solving goal should permeate the instruction of elementary school teachers in all mathematics content courses and in all education courses. They also felt that problem solving could best be "defined" by quoting authorities who have devoted much of their professional life to the study of the problem-solving processes. The quotations below are rather extensive, but the reader should find them instructive.

George Polya. MATHEMATICAL DISCOVERY, Vol. 1. John Wiley & Sons, 1961.

Solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable. Solving problems in the specific achievement of intelligence, and intelligence is the specific gift of mankind; solving problems can be regarded as the most characteristically human activity. (Preface v)

Solving problems is a practical art, like swimming, or skiing, or playing the piano: you can learn it only by imitation and practice. ...if you wish to learn swimming you have to go into the water, and if you wish to become a problem solver you have to solve problems. (Preface v)

If you wish to derive the most profit from your effort, look out for such features of the problem at hand as may be useful in handling the problems to come. A solution that you have obtained by your own effort or that you have read or heard, but have followed with real interest and insight, may become a pattern for you, a model you can imitate with advantage in solving similar problems. (Preface v)

I wish to call heuristic...the study of means and methods of problem solving. (Preface v)

To have a problem means: to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim. To solve a problem means to find such action. A problem is a great problem if it is very difficult; it is just a little problem if it is just a little difficult. Yet some degree of difficulty belongs to the very notion of a problem: where there is no difficulty, there is no problem. (Page 117)

Progressive Education Association. MATHEMATICS IN GENERAL EDUCATION. D. Appleton-Century Co., Inc., 1940.

Reflective thinking originates when an individual feels a sense of perplexity, which leads to the identification and formulation of a problem. Further progress depends upon the development of a tentative hypothesis (or hypotheses) based upon the data at hand, and this in turn often suggests the need for additional data. If the data tend on analysis to support the orginal hypothesis, this hypothesis is taken as a conclusion, and the problem is considered solved. On the other hand, the data may eventually be found to weaken the hypothesis or make it wholly untenable, which leads to a similar examination of alternative hypotheses until a satisfactory solution is reached. (Page 40)

Excerpts from R. L. Thorndike article in LEARNING AND INSTRUCTION, Forty-Ninth Yearbook, National Society for the Study of Education (NSSE)

Problems are of all levels of complexity, scope, and subtlety. ...but they all have in common three elements....

1. The individual is oriented toward a particular objective and motivated to reach it. He has an end in view.

2. Progress toward the objective is blocked.

3. Available, habitual response patterns are not adequate to permit the individual to surmount the obstacle and proceed toward the objective. (Page 193)

It is perhaps worthwhile to make a distinction between two types of problems which arise in life, which we may call (a) practical problems and (b) intellectual problems. Type (a) is motivated by a need to act; type (b) by a need to understand. (Page 195)

A familiar analysis, owing its main debt to Dewey[1], analyses the process into five phases. These phases represent a logical sequence of steps to be carried out in an idealized process of problem solution. The phases may be defined as follows:

1. Becoming aware of a problem. The route to some objective is blocked, routine behavior is not directly successful, and the individual realizes that a problem exists.

2. Clarifying the problem. The problem, sensed at first only in general terms, is made more sharp and specific in terms of just what end is to be achieved and just what is known or what resources are available.

3. Proposing hypotheses for solution of the problem. Specific proposals are suggested and elaborated for dealing with the problem situation.

4. Reasoning out implications of hypotheses. Bringing together the hypothesis and the relevant facts which are known to him, the individual reasons out what follows from the hypothesis which he is considering.

5. Testing the hypothesis against experience. The conclusions which follow from the hypothesis are tested against known facts or by experiment and the gathering of new facts, to see if the conclusions are valid and the hypothesis is supported.

[1]John Dewey, HOW WE THINK, pp. 106-15. Boston: D.C. Heath & Co., 1933. (Pages 196-197)

We must recognize, however, that this is an analysis to help our understanding of the problem-solving enterprise, rather than a description of any individual actually carrying through the solution of a problem. The analysis is neat, logical, and sequential. Actual behavior in response to a problem situation is often confused, illogical, and disorderly.

Furthermore, each problem-solver and each problem to be solved has its own individual characteristics. Diversity rather than uniformity is the rule in the attack upon problem situations. We do not find the problem-solver going neatly and logically through the sequence of steps outlined above. Rather, he jumps around, often starting in the middle, returning then to the initial steps, moving back and forth between hypothesis, problem clarification, appraisal of implications, and hypothesis again. Some of the phases outlined may fail to appear, as when a hypothesis is put into action without previously thinking through what it means or implies.

William A. Brownell. "Problem Solving," THE PSYCHOLOGY OF LEARNING, Forty-First Yearbook, NSSE, 1942.

...problem solving refers (a) only to perceptual and conceptual tasks, (b) the nature of which the subject by reason of original nature, of previous learning, or of organization of the task, is able to understand, but (c) for which at the time he knows no direct means of satisfaction. (d) The subject experiences perplexity in the problem situation, but he does not experience utter confusion. From this he is saved by the condition described above under (b). Then, problem solving becomes the process by which the subject extricates himself from his problem, a statement which is deliberately left vague at this point in the chapter.

Defined thus, problems may be thought of as occupying intermediate territory in a continuum which stretches from the 'puzzle' at one extreme to the completely familiar and understandable situation at the other. In the case of puzzles, the nature of the task may vary from the wholly novel (which the learner has no means of solving or escaping) to the slightly known (in which the learner may recognize vaguely where to work, though he does not know what to do). His present experience is one of bewilderment, and this is brought on by the strangeness of the situation and its lack of meaning to him. If he is successful in solving the puzzle, his success is the result of accident, and the solution is not likely to be retained or to be transferable to other similar puzzle situations. In the case of completely familiar and understandable situations, no problem exists, as is true of puzzles, for the reason that the learner has available satisfactory responses which have been habituated, and there is no uncertainty as to procedure. (Page 416)

The distinction between problems and other situations lies in the peculiar relationship which exists between the learner and his task. (Page 416)

The problems offered by the school should depend for their solution, not on guessing and habituation of chance successes, but on progressive growth in understanding. (Page 417)

All schools of psychology accept the fact that behavior in a problem situation is no hit-or-miss, no haphazard affair; but that, instead, it has a directed character. (Page 422)

The importance of these concepts (insights and trial and error) is probably greater for problem solving than for other types of learning. (Page 423)

Few investigators have sought to get beneath the errors (in problem solving) to ascertain their causes. Yet, it is this latter knowledge about errors which is of most vital importance for the improvement both of initial and remedial instruction. (Page 427)

Nevertheless, with all their limitations, Piaget's studies seem to provide the most illuminating single description of the way in which children attain power in problem solving. (Page 428)

Katona has demonstrated, first, that problem solving which is based upon understanding is superior to 'problem solving' based upon memorization; second, that understanding is a matter of degree, that varying degrees of understanding affect problem solving differently, and that the degree of understanding engendered is a function of the kind of instruction given; and, third, that form of instruction which enables the learner best to organize his previous experiences or learnings is to be preferred to other kinds. (Pages 436-7)

Teaching should start with whatever technique the child uses proficiently and should guide him in the adoption and use of steadily more mature types of problem solving. (Page 439)

There is reason to believe that meanings and understandings are most useful in problem solving when they have themselves been acquired through the solving of problems. (Page 439)

To be most fruitful, practice in problem solving should not consist in repeated experiences in solving the same problems with the same techniques, but should consist in the solution of different problems by the same techniques and in the application of different techniques to the same problems. (Page 439)

Part of real expertness in problem solving is the ability to differentiate between the reasonable and the absurd, the logical and the illogical. Instead of being 'protected' from error, the child should many times be exposed to error and be encouraged to detect and to demonstrate what is wrong, and why. (Page 440)

Obviously, the above writers consider problem solving to be something more than solving textbook story or work problems, important as these "problem-solving" skills may be. The types of learning experiences which mathematics teachers should provide are those in which students do not have a ready way of reaching the solutions. Such experiences provide settings in which students can improve their problem-solving abilities. Apparently, for most students such problem-solving experiences do not automatically mean improvement in their problem-solving ability. Recently a study[1] was made which indicated that attention given in instruction to heuristics (problem-solving strategies) enabled students to use these strategies in seeking

[1] Pereira-Mondoza, Lional. The Effect of Teaching Heuristics on the Ability To Solve Novel Mathematical Problems. The University of British Columbia. July 1975.

solutions to novel mathematical problems. If the results of the study are generally true, then much more attention should be devoted to problem-solving strategies in mathematics teacher education courses.

Nearly twenty years ago certain mathematics educators in Oregon prepared a list of abilities that competent problem-solvers most likely would possess. This list is included below with the hope that it might prove helpful to mathematics teacher educators as they help their students focus their attention on the problem solving process. It might also be used by classroom teachers as they plan activities for their students. However, it should be kept in mind that problem solving is the most complex behavior that a human being can acquire. Therefore, any listing that might be made and any order implied by the listing is a gross simplification.

Recognizing and Defining the Problem

1. Recognize problem situations in which mathematics can be used in obtaining answers.

2. Diagram, dramatize, and/or construct models of problem-solving situations (especially applicable to story problems.)

3. State problem in own words.

4. Recognize problems in which there is not enough information given for solving the problem (insufficient data).

5. Recognize in those problems with insufficient data what data needs to be obtained before the problem can be solved.

6. Identify the kind of data needed if the problem is to be solved (relevant data).

7. Display a reasonable understanding of the problem situation before attempting to solve the problem (meaning of terms are known and other concepts involved are understood).

8. Formulate mathematical statements that describe problem situations (including story problems).

Estimating and Predicting

9. Make predictions which are neither overcautious nor overimpulsive.

10. Judge the limits between which answers should lie if the answer is to be reasonable.

Collecting, Organizing, and Presenting Data

11. Ignore or discard data that is not needed in the solution of a problem.

12. Use sources of information effectively for the purpose of finding the needed data.

13. Display a questioning attitude toward the data as to its reliability, accuracy, and representativeness.

14. Present data in a well-organized way so that it can be easily read.

15. Present problem solutions neatly and in a well-organized way so others may easily follow the reasoning involved.

Plan of Attack

16. Find alternative methods for solving the same problem.

17. Select the best mathematical procedure(s) that will obtain an answer to a given problem.

18. Change a problem whose solution is not known into an analogous problem whose solution is known.

19. Work back from a tentative conclusion to the given data in the hope of finding a technique for solving the problem.

20. Point out those cues in the problem situations (especially applicable to story problems) which suggest certain methods of attack.

21. Find in textbooks and other library resources new techniques for solving a problem.

22. Demonstrate mathematical reading skills which will allow him to understand textbook explanations.

Interpretation and Application

23. Recognize relationships and trends (if any exist) suggested by the data collected.

24. Translate mathematical language into language of common usage.

25. Interpret the answers of mathematical operations by referring back to the original problem setting.

26. Determine when a problem has a reasonable answer.

27. Recognize those actual problem situations which could make use of acquired mathematical skills and techniques.

28. Invent problem situations which can make use of known techniques.

Search for New Problems

29. Invent new problems by varying in some way the "problem" whose solutions are known.

30. Generalize a solution to a problem so as to include related problems.

PEDAGOGY
ELEMENTARY SCHOOL MATHEMATICS

PREAMBLE - PEDAGOGY AND CONTENT

In an effort to emphasize the importance of both content and methodology, it was felt that it would be useful to separate topics into two sections, one focusing on content (the mathematical knowledge of an elementary teacher) and the other focusing on pedagogy (the ability to transmit that knowledge to children). However, the separation in this paper should not be construed to suggest that mathematics and pedagogy should be separated either in our programs or in our evaluation of elementary teachers. The mathematics should be taught in a way which will illustrate by example the pedagogical skills elementary teachers are expected to have. Similarly, the pedagogical skills should be taught in relation to the various mathematical areas.

In both the mathematical and pedagogical courses, it is intended that an inquiry, problem solving approach will be emphasized. Elementary teachers should have a sense of the interrelationships between the various areas of mathematics and the relationships of mathematics to other subjects. They should be aware of the historical and cultural aspects of mathematics. Finally, interest in applications of mathematics should never be allowed to smother the development of an aesthetic appreciation of the subject.

PSYCHOLOGY OF LEARNING

EXPECTANCY	INDICATOR SAMPLES
1. Demonstrate ability to apply various theories of instruction of children and the psychology of learning.	1.1 Develop a lesson plan based on concrete learning experiences.
	1.2 Describe some activities which would help determine a child's developmental readiness for some topic such as measurement.
	1.3 Choose a topic and give examples of ways to introduce it to motivate learning.
	1.4 Describe a plan of instruction with types of rewards which will provide positive reinforcement.
	1.5 Develop a procedure for determining whether a child is learning inductively or deductively in a specific situation.
2. Demonstrate ability to provide rationale for teaching strategies	2.1 For a particular lesson be able to explain, in terms of learning theories such as Piaget's, Bruner's or Skinner's why the strategies used were chosen for that lesson.

DIAGNOSTIC-PRESCRIPTIVE TECHNIQUES

EXPECTANCY	INDICATOR SAMPLES
1. Demonstrate knowledge of techniques for determining a child's readiness for a new topic.	1.1 List the entry skills for some topic and make a plan for determining whether a child has the desired skills.
2. Show ability to ascertain the mathematical knowledge of a student.	2.1 Develop a procedure for determining the concepts and skills mastered by a child recently transferred to your school.
3. Demonstrate ability to organize a topic in terms of a hierachy or sequence of components.	3.1 For some topic, describe a series of learning activities which develop the topic, one concept or skill at a time.
4. Demonstrate ability to prescribe for correction of errors made by students who are working on a topic.	4.1 Determine lack of understanding pupils have by analyzing errors they make.

Example:

$$\begin{array}{r} 42 \\ -\underline{17} \\ 35 \end{array} \qquad \begin{array}{r} 95 \\ -\underline{23} \\ 72 \end{array} \qquad \begin{array}{r} 81 \\ -\underline{57} \\ 36 \end{array}$$

5.	Demonstrate knowledge of techniques for determining a child's mastery of a topic.	5.1	Describe a desired level of achievement for some topic and make a plan for determining whether a child has attained that level.
6.	Demonstrate ability to provide for a wide range of learning abilities.	6.1	For some topic, describe a variety of strategies and activities suitable for slow learners.
		6.2	Select additional activities for those students who master a topic quickly.

RESOURCES AND MATERIALS

	EXPECTANCY		INDICATOR SAMPLES
1.	Demonstrate familiarity with mathematics education literature.	1.1	Locate several journal articles related to teaching some specific topic.
		1.2	For some topic, develop a list of resource books to supply a variety of activities.
2.	Demonstrate ability to use a variety of manipulatives, such as the abacus, attribute blocks, bean sticks, the geoboard, measuring devices, and number rods.	2.1	Plan a series of activities using a given manipulative.
		2.2	For some topic, develop a plan for using a variety of manipulatives.
3.	Demonstrate ability to use a teacher's manual.	3.1	Develop a lesson plan based on a textbook and its teacher's manual.

TEACHING STRATEGIES

	EXPECTANCY		INDICATOR SAMPLES
1.	Demonstrate familiarity with various forms of classroom organization.	1.1	In an individualized program described how to introduce topics, set goals for each child, keep records and evaluate progress.
		1.2	Design group activities which provide opportunities to individuals within the group to function at their own level.
		1.3	Develop a plan for managing the instruction of three or more ability or interest groups within the classroom.
2.	Show the ability to plan and use a variety of modes of instruction.	2.1	Select several games and puzzles suitable for drill on some skill.
		2.2	Design and construct a learning center for either introducing or extending a topic.
		2.3	Plan a laboratory investigation to develop a skill such as estimation.
		2.4	Plan a unit in another subject field which requires extensive applications of mathematic
		2.5	Plan a guided discovery lesson.

CONTENT SECTION,
ELEMENTARY SCHOOL MATHEMATICS

Mathematics for elementary school teachers should be taught in such a way as to exemplify sound pedagogical principles including a problem-solving approach to instruction. Good instruction in mathematics also implies that the students sense the interrelationships between topics and branches of mathematics. The listing of mathematical topics which follows is done as a matter of convenience and should not be interpreted as being a scope and sequence for mathematics courses for elementary school teachers.

Sets, Numbers and Numeration

Program Goal

To provide a background in sets and numbers, and to develop an awareness of problems most likely encountered in teaching related concepts to children.

EXPECTANCY	INDICATOR SAMPLES
1. The teacher can use correctly the various set notations found in elementary mathematics textbooks.	1.1 Given a set description in a set-builder notation, roster notation or verbal description, verbalize the symbols and identify members in the set.
	1.2 Given a set description in a set-builder notation, roster notation or verbal description, represent the set in the other two conventions.
2. The teacher can demonstrate the knowledge of cardinal number as a property of equivalent sets.	2.1 Construct more than one, and also be able to identify any, of the 1-1 correspondences between a pair of equivalent sets when the cardinality of each aet is greater than 1.
	2.2 Identify the smaller (or larger) of two given whole numbers using the notion of a 1-1 correspondence between one set and a proper subset of another set.
3. The teacher will understand intersections, unions, cartesian products, complements and sets.	3.1 Given two nonempty sets A and B, both in roster form, display elements of A x B, $A \cup B$, $A \cap B$.
	3.2 Illustrate the relationship and identify restrictions between whole number operationa and the set concepts of intersection, union, cartesian product, complement and set difference.
4. The teacher can demonstrate an understanding of the Hindu-Arabic system and other similar place value numeration systems.	4.1 Describe relationships among number, numeral, and various expanded forms, and exponential notation.

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EXPECTANCY	INDICATOR SAMPLES
	4.2 Use correctly and explain clearly: a) a non-place value system, b) a place value system other than Hindu-Arabic.
	4.3 Use manipulative materials and/or models to illustrate the place value concept in the Hindu-Arabic numeration system.
5. The teacher can use different techniques for developing the meaning of fraction and equivalent fractions.	5.1 Use models such as the following to illustrate fractions: Sets, arrays, Cuisenaire rods, Dienes blocks, etc., area (region), linear (number line), geoboards, chips, cubes, tangrams and drawings.
	5.2 Express the given numbers in both decimal and fractional form.
6. The teacher can describe the relationship between whole numbers, intergers, rationals, and reals.	6.1 Give some reasons for the invention of zero, negative numbers, rational, and irrational numbers.

OPERATIONS

Program Goal:

To provide background in basic operations and an introduction to alternative algorithms.

EXPECTANCY	INDICATOR SAMPLES
1. The teacher can demonstrate an understanding of addition and subtraction (with or without regrouping), multiplication and division (with or without remainder), of rational numbers.	1.1 Demonstrate addition and subtraction (with or without regrouping), multiplication and division, (with or without remainder) using physical materials such as a Set Model (bundle of sticks, chips, bean sticks, Dienes blocks, bead frames, cubes, etc.) and a Measurement Model (number line, linear model, geoboard, cubes, etc.)
	1.2 Use grouping symbols (parentheses and brackets) to indicate the conventional order of operations.
2. The teacher can demonstrate an understanding of more than one algorithm for each of the four arithmetic operations.	2.1 Add, subtract, multiply and divide numbers using algorithms such as: The standard algorithms, doubling and adding, the lattice method, the Russian Peasant method, successive subtraction, scratch method, etc.
	2.2 Use a calculator in making computations.

EXPECTANCY		INDICATOR SAMPLES
3. The teacher can demonstrate an understanding of addition, subtraction, multiplication and division of rational numbers in fractional and decimal form.	3.1	Use set and measurement models to demonstrate addition, subtraction, multiplication, and division of rational numbers in fractional or decimal form.
	3.2	Demonstrate how the operations of rational numbers extend those of the whole numbers.
	3.3	Can use the number line, the thermometer and/or other devices for illustrating the rules of operations with integers.
4. The teacher can demonstrate and apply the following properties: Closure (+,x) Commutative (+,x) Associative (+,x) Identity(+,x) Distributivity (x/+ and +/x) Inverses for whole integers and rational numbers.	4.1	Use set and measurement models to demonstrate the properties of whole and rational numbers.
	4.2	Identify with examples which of these properties hold (and which do not) for the operations subtraction and division.
5. The teacher can demonstrate some familiarity with operations other than addition, subtraction, multiplication, and division.	5.1	Give an example of an operation such as square root, squaring a number, or taking an average; and describe one or more of its properties.

NUMBER SENTENCES

Program Goal:

To develop an understanding of elementary mathematical sentences and their applications.

EXPECTANCY	INDICATOR SAMPLES
1. The teacher can use correctly conventional mathematics symbols found in contemporary elementary math texts.	1.1 In written and spoken communication, use correctly the following as verbs: $=, \neq, <, \nless, >, \ngtr$
	1.2 Given whole number sentences or expressions using parentheses, illustrate the use of the parentheses to override the conventional order of operations.
2. The teacher can correctly use variables and solve elementary open sentences involving whole and rational numbers.	2.1 Given open sentences (both equalities and inequalities), find the solution set from the whole numbers, integers or rational numbers.
3. The teacher is able to translate the given problem situation into a mathematical sentence or model, find a solution for the model and reinterpret the mathematical solution back into the context of the problem situation.	3.1 Given a mathematical problem situation (appropriate for children), write an open sentence which models the problem.
	3.2 Given an open sentence, describe a problem situation (appropriate for children) from which the open sentence could be derived.
4. The teacher understands and can correctly use ratio and proportions.	4.1 Use proportions to convert common fractions to their equivalent percents.
	4.2 Give real world examples of the use of ratio and proportion problems that might occur in industry, home or leisure activities.

NUMBER THEORY

Program Goal:

To develop an understanding of elementary number theory and its uses.

EXPECTANCY	INDICATOR SAMPLES
1. The teacher is familiar with the common classifications of positive whole numbers.	1.1 Given the set of positive whole numbers less than 100, identify the evens, odds, units, primes, and composite numbers.
	1.2 Demonstrate a physical model of prime and composite numbers.
2. The teacher can demonstrate an understanding of the Fundamental Theorem of Arithmetic.	2.1 Demonstrate at least one process that illustrates the unique prime factorization principle of positive integers greater than one.
	2.2 Given two or more natural numbers, each less than 100, demonstrate processes which will yield their greatest common factor (GCF) and least common multiple (LCM).
3. The teacher can determine factors of whole numbers less than 100 mentally.	3.1 Using tests of divisibility, identify given numbers as being divisible by 2, 3, 4, 5, 6, 8, 9, and 10.
	3.2 Given a set of fractions, reduce each one to lowest terms.
4. The teacher can determine whether or not an answer to a quantitative problem is reasonable.	4.1 Given appropriate numbers, estimate sums, products, quotients, and differences.
	4.2 Perform operations, with powers of 10, mentally.
5. The teacher can demonstrate conceptualizations for large numbers ($\leq 10^9$) and small numbers ($\geq 10^{-3}$) and can generalize patterned sequences.	5.1 Offer models which demonstrate the magnitude of large numbers and small numbers.
	5.2 Given some sequence or numerical pattern whose description lies within the range of the student's supposed experience, generalize the sequence or describe the pattern.

EXPECTANCY	INDICATOR SAMPLES
6. The teacher can demonstrate an understanding of modular systems and is able to apply such to practical situations.	6.1 Complete the tables for addition and multiplication in some modular system and discuss commutativity, associativity, identity, inverses under both operations and distributivity.
	6.2 Describe real world problems that can be solved easily using modular systems. (Possibly modular arithmetic could be related to problems involving a clock.)

LOGICAL THINKING

Program Goal:

To develop an understanding of the uses of precise language and logical thought processes.

Expectance		Indicator Samples
1. The teacher will make qualitative discriminations.	1.1	Recognize the various attributes of items in a given set, and describe the set in terms of the attributes of its elements.
	1.2	Divide a set into subsets according to given attributes.
	1.3	Order the elements of a set according to prescribed conditions.
2. The teacher will understand quantified and open sentences.	2.1	Determine the truth value of statements with quantifying words such as "all," "each," "at least one," "some," "one and only one," etc.
	2.2	Recognize open sentences and determine domains and truth sets of the variables in these sentences.
	2.3	Illustrate the meaning of quantified statements using Venn diagrams.
3. The teacher will understand and use correctly the logical operations of disjunction, conjunction, negation, and implication.	3.1	Correctly interpret and use connectives "or," "and," "if,...,then," "if and only if," and "not."
	3.2	Write in simplified form the negations on sentences with one or more connectives.
	3.3	Illustrate the meaning of sentences with connectives using attribute blocks, sets, or truth tables.
4. The teacher will recognize valid and invalid reasoning, and will correctly use simple deductive and inductive reasoning.	4.1	Recognize and illustrate uses of the more common deductive reasoning patterns.
	4.2	Observe patterns, form and structure of given or observable information, and arrive at plausible conclusions from this information.
	4.3	Test the validity of arguments using truth tables, Venn diagrams, counterexamples, etc.

GEOMETRY AND MEASUREMENT

Program Goal:

To develop an understanding of the language and concepts of elementary geometry and measurement, including SI metric, and their applications.

EXPECTANCY		INDICATOR SAMPLES
1. The teacher can read and correctly interpret common symbols for geometric objects.	1.1	Given symbols such as those below, draw a good sketch of each: $\angle MPQ$ $\overleftrightarrow{MP} \parallel \overleftrightarrow{CQ}$ $\overline{RV}$ $\overline{ND} \perp \overline{XY}$ $\overrightarrow{VW}$ $\triangle MPN \cong \triangle XYZ$
2. The teacher can sketch and label common geometric figures with some accuracy.	2.1	Sketch and label distinguishing characteristics of such figures as a quadrilateral, a parallelogram, a rectangle, a square, a rhombus, and a trapezoid.
	2.2	Sketch three dimensional objects such as a rectangular prism, a pyramid, a cylinder, a cone, a cube, and a sphere.
3. The teacher can select and use the appropriate tools of geometry in constructions and in making measurements.	3.1	Given a compass and a straightedge, bisect a given angle and a line segment, copy an angle, construct perpendicular lines, and construct a square.
	3.2	Given a protractor and straightedge, measure angles, and draw angles of a required measure.
	3.3	Given a measurement instrument (ruler, protractor, scale balance), measure an object to the nearest specified unit.
	3.4	Given a measuring instrument, state the greatest precision possible and determine the relative error for a measurement resulting from use of the given instrument.
4. The teacher can estimate distances, area, weight, and volume in SI metric units.	4.1	Given a line segment or an object less than 20 cm long, estimate its length in cm (within a given tolerance).
	4.2	Given a simple closed curve, find upper and lower bounds of the perimeter and area within a given tolerance.
5. The teacher can demonstrate understanding of direct measurement.	5.1	Given suitable objects, but no standardized measuring device, select an appropriate unit and demonstrate how it may be used to find the length, area, volume, or angle measure.

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EXPECTANCY		INDICATOR SAMPLES
	5.2	Given an object to measure and a collection of measuring devices, select the appropriate device(s) and measure the object to the nearest unit of measure.
6. The teacher can demonstrate an understanding of the approximate nature of measurements and the extent.	6.1	Given the length and width of a rectangle to the nearest centimetre, state the upper and lower bounds for its perimeter and area.
	6.2	Use correctly terms such as greatest possible error, significant digits, relative error and precision in describing the reliability of measurements, and calculations with measurements.
7. The teacher can use principles of indirect measure.	7.1	Compute a good approximation of the height of a given building, flag pole, cliff or other tall object without actually measuring the object.
	7.2	Compute a good approximation of the number of marbles, Ping-Pong balls, or other objects needed to fill a given room or space.
8. The teacher can describe how to find the area of any polygon.	8.1	Demonstrate how to find the area of a parallelogram, triangle, and trapezoid assuming the area of a rectangle.
	8.2	Given any polygon, describe a process for finding its area.
9. The teacher can describe how to find the volume and area of the surface of common three-dimensional objects.	9.1	Find the volume and surface area of a rectangular prism, pyramid, cylinder, cone, and sphere.
10. The teacher can use correctly the metric system (SI) of measurement.	10.1	Give the meaning of the prefixes milli-, centi-, and kilo-.
	10.2	Identify the metric basic units for length mass, volume and temperature as well as the interrelationships among these units.
11. The teacher can use the principles of measurement in practical situations.	11.1	Given a road map, topological map, or globe, and appropriate information, locate various points and determine distances.
	11.2	Given a household problem involving measurement, select the appropriate measuring instruments and solve the problem.